Beyond Algebra

Topics in Precalculus, Discrete Mathematics and Calculus Using the TI-84 and TI-89

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Trigonometry on the TI-83/4

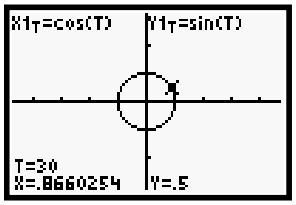
When working with angles and trigonometric functions, it is important to note whether your handheld is in **Degree** mode or in **Radian** mode. Home screen calculations, angle conversions and graphs will be affected by this mode setting.

Exploring The Unit Circle

1. In Degree mode, graph the parametric equations $x = \cos(t),\ y = \sin(t)$.

- How should the window be set?
- Why?

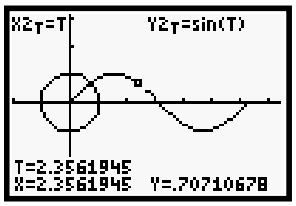
2. Use ZStandard followed by ZDecimal.



- What effect does each of these [ZOOM] features have?
- Explore the points of the graph using the [TRACE] feature.
- Switch to Radian mode and repeat. See Figure 2.

The Unit Circle And The Sine Graph

Use Radian, Parametric, and Simultaneous graphing modes.

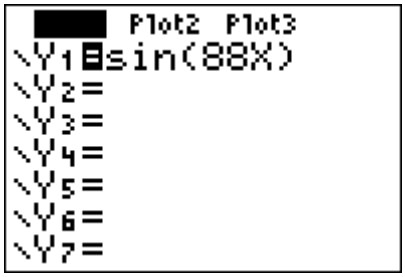


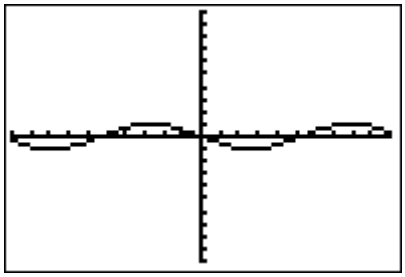
- Graph the unit circle and the sine graph together in one window.
- What Window settings should be used?
- Explore the points of the graph using the [TRACE] feature.

A Misleading Graph

Mode: Func, Radian. Graph ***Y1=sin(90x)*** using ZStandard.

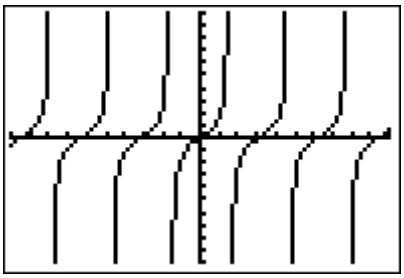
- What does the period of this function appear to be? What is the actual period?
- Graph $Y2 = \sin(x\sqrt{2})$
- Use ZBox to zoom in around the first arch of the curve





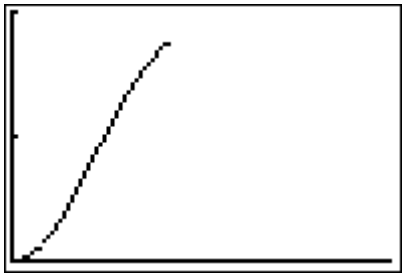
New Graphing Algorithm (TI-84 OS 2.3 and above)

- Graph ***y1=tan(x)*** in ZStandard
- Change xRes to 2 in WINDOW



The 'Birthday Problem' and the Logistic function

The probability that at least two people in a group of *n* people have the same birthday (month and day) is given by the formula $P(n) = 1 - \frac{365 \text{ nPr } n}{365^n}$.

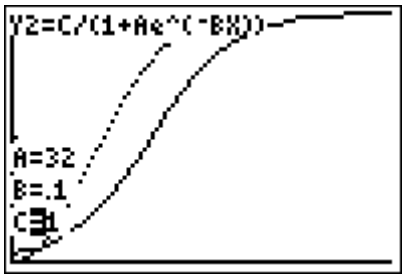


- Graph y1=1-(365 nPr x)/365^x in x[0,94], y[0,1]. Why does the graph stop at *x*=39?

- Use the TRANSFRM app to fit a logistic function to this partial graph. But First:
  TRANSFRM can only graph one 'interactive' function, but it can graph it over a StatPlot, so

- In the Stat Editor, clear out two lists (list1 and list2) and, in the first list enter the sequence {1,2,3,…,39}(use the *seq()* function!)
  In the second list compute Y1(list1).
  Set up a Stat Plot to display a scatterplot using L1 and L2 and the 'pixel' mark in the same viewing window as before.

- Now, invoke TRANSFRM, enter the logistic function $y1 = \frac{c}{1 + ae^{-bx}}$ and play around to find values for a, b, and c that cause the logistic function to 'fit' the probability function. Is there any significance to the values you find for these parameters? Start with a=32, b=.1, c=1



Discrete Mathematics

Combinations

The current version of Powerball has 5 boards. Each board contains two sets of numbers, one set from 1 to 49 and one set from 1 to 42. Each play consists of picking 5 numbers from the first set of 49 and 1 number, the Powerball, from the second set of 42.

Below is a representation of a Powerball lottery card. Pick 5 numbers on the top portion and 1 number in the lower portion in each of the 5 columns of the card.

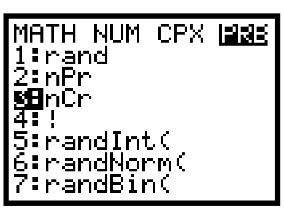
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
10 11 12 13 14 15 16 17	10 11 12 13 14 15 16 17	10 11 12 13 14 15 16 17	10 11 12 13 14 15 16 17	10 11 12 13 14 15 16 17
18 19 20 21 22 23 24 25	18 19 20 21 22 23 24 25	18 19 20 21 22 23 24 25	18 19 20 21 22 23 24 25	18 19 20 21 22 23 24 25
26 27 28 29 30 31 32 33	26 27 28 29 30 31 32 33	26 27 28 29 30 31 32 33	26 27 28 29 30 31 32 33	26 27 28 29 30 31 32 33
34 35 36 37 38 39 40 41	34 35 36 37 38 39 40 41	34 35 36 37 38 39 40 41	34 35 36 37 38 39 40 41	34 35 36 37 38 39 40 41
42 43 44 45 46 47 48 49	42 43 44 45 46 47 48 49	42 43 44 45 46 47 48 49	42 43 44 45 46 47 48 49	42 43 44 45 46 47 48 49
1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7
8 9 10 11 12 13 14	8 9 10 11 12 13 14	8 9 10 11 12 13 14	8 9 10 11 12 13 14	8 9 10 11 12 13 14
15 16 17 18 19 20 21	15 16 17 18 19 20 21	15 16 17 18 19 20 21	15 16 17 18 19 20 21	15 16 17 18 19 20 21
22 23 24 25 26 27 28	22 23 24 25 26 27 28	22 23 24 25 26 27 28	22 23 24 25 26 27 28	22 23 24 25 26 27 28
29 30 31 32 33 34 35	29 30 31 32 33 34 35	29 30 31 32 33 34 35	29 30 31 32 33 34 35	29 30 31 32 33 34 35
36 37 38 39 40 41 42	36 37 38 39 40 41 42	36 37 38 39 40 41 42	36 37 38 39 40 41 42	36 37 38 39 40 41 42

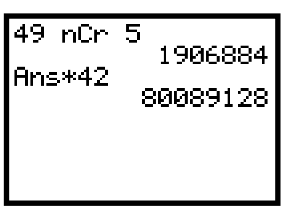
How many possible way of selecting the 5 numbers and the 1 powerball number are possible?
What is the probability of winning?
If you spend $5 for this card, what is your expected value?

The number of combinations of n objects taken r at a time is denoted ${}_nC_r$ and is given by

$${}_nC_r = \frac{n!}{r!(n-r)!}.$$

The ${}_nC_r$ is located in the **MATH <PRB> 3:nCr**.
Multiply the number of ways to choose 5 from 49 on the top half times the 42 ways to chose one number on the bottom half.





Exercises

1. The previous version of Powerball had 45 numbers in the top half and 45 numbers in the lower half. Players had to select 5 from the top and 1 from the bottom. How many ways were possible in the original version of Powerball?

2. State Lotteries

a) The state of Kansas has a game called *Cash4Life*. In order to win $1000 a week for life, the player must select 4 numbers between 00 and 99. How many ways are possible to pick these four numbers?

b) *Kansas Cash* requires the player to select 6 numbers between 1 and 35 to win a minimum jackpot of $100,000. How many ways are possible for the player to pick these six numbers?

c) *Pennsylvania Cash* is played by choosing 5 numbers from 39. How many ways are possible to select the 5 numbers?

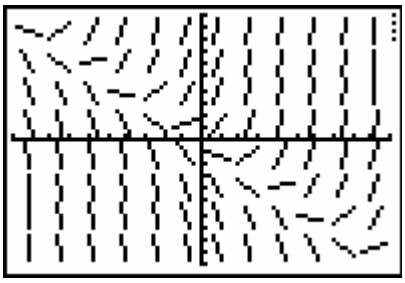
d) The *Keystone Jackpot* is played by choosing 6 numbers from 33 on the top half and 1 from 33 on the bottom half. How many ways of picking these numbers are possible?

Calculus on the TI-83/4: Slope Fields and Euler's Method

A TI-83/4 Slope Field Program

An important topic in Calculus, the slope field illustrates the direction of a function at points in the plane by drawing representative tangents at various places. Use this program with "FNINTY1" below to generate graphical solutions to differential equations. Enter a differential equation (in terms of x and y) into Y1 and run SLOPEFLD.
(example, for $dy/dx = x+y$, enter Y1 = X + Y)

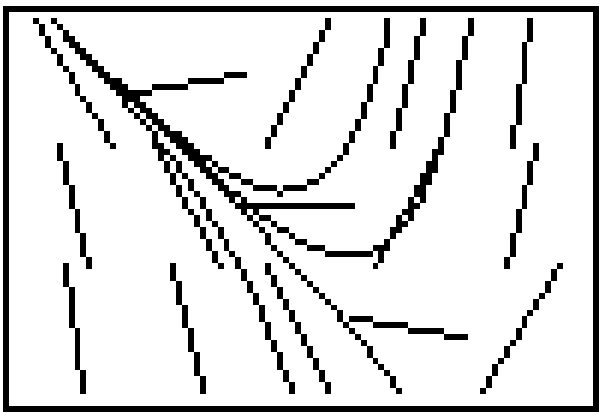
```
Program:SLOPEFLD
Func:ClrHome
Disp "BE SURE TO"
Disp "ENTER DY/DX="
Disp "F(X,Y) IN"
Disp "TERMS OF X"
Disp " AND Y IN "
Disp " THE Y= EDITOR"
Disp " PRESS ENTER"
Disp "TO BEGIN"
Pause
Lbl 1
ClrHome
Input "NO. X-VALUES ",W
Input "NO. Y-VALUES ",L
(Ymax-Ymin)/L→V
(Xmax-Xmin)/W→H
ClrDraw
FnOff
Ymin+V/2→Y
For(R,1,L)
  Xmin+H/2+.000001→X
  For(C,1,W)
    Y1→M
    -M*H/2+Y→S
    M*H/2+Y→T
    If abs((T-S)>V
    Then
      Y+V/2→T
      Y-V/2→S
      (T-Y)/M+X→Q
      (S-Y)/M+X→P
    Else
      X-H/2→P
      X+H/2→Q
    End
    Y→F
    Line(P,S,Q,T)
    F→Y
    X+H→X
  End
  Y+V→Y
End
```



…and FNINTY1...

Enter your integrand (a dif-eq in *x* and *y, same as in SLOPEFLD above*) in Y1, set an appropriate viewing window, then run this program that uses *Euler's Method* to draw an antiderivative. From the menu, choose a 'speed factor': 1, 2, or 3 (higher is faster). After the function is graphed, Trace is activated to choose a starting point for integration. Select a point and press [ENTER]. An indefinite integral is plotted, then back to Trace to select another starting point. Press [ON] to break out of the "infinite loop" caused by the statement *While 1*.

```
Program:FNINTY1
     :MENU ("CHOOSE ΔX INC.","1ΔX",1,"2ΔX",2,"3ΔX",3)
     :LbL 1: 1→D: Goto 4
     :Lbl 2: 2→D: Goto 4
     :Lbl 3: 3→D
     :Lbl 4
     :ClrDraw
     :While 1
       :Trace
       :X→A:Y→B
       :X→J:Y→K
       :For(X,A,Xmin,⁻DΔX)
         :Line(J,K,X,Y)
         :X→J:Y→K
         :Y-DY₁ΔX→Y
       :End
       :Ø→Y:A→J:B→Y:Y→K
       :For(X,A,Xmax,DΔX)
         :Line(J,K,X,Y)
         :X→J:Y→K
         :DY₁ΔX+Y→Y
       :End
     :End
```

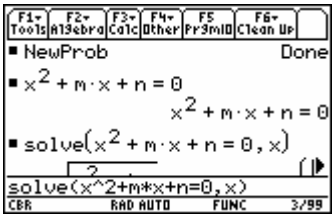


SLOPEFLD and FNINTY1 at work

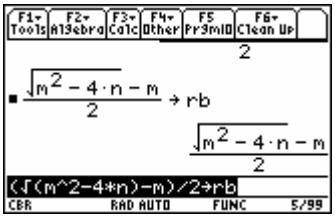
Computer Algebra Systems (the TI-Voyage 200 and the TI-89)

Step-by-Step

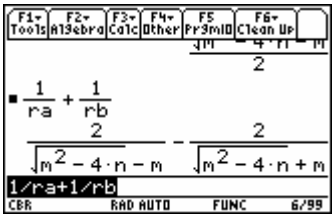
1. Find, in terms of m and n, the sum of the reciprocals of the roots of $x^2 + mx + n = 0$



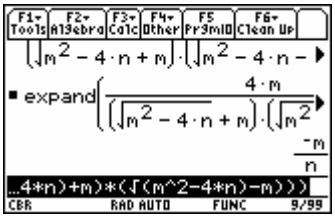
First, start a "NewProb" ([F6] 2:)
Enter the equation to solve *(for later use)*
Use [F2]: **solve(** to solve for *x*: First, put the command down, then go up (▲)and retrieve ([ENTER]) the equation, then type a *'comma'* and the letter *x*, close parentheses, and press [ENTER] to evaluate the command.



Use copy and editing tools to store the two roots in the variables *ra* and *rb*. Follow me.



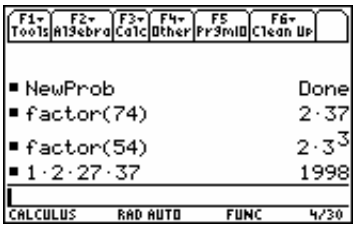
Evaluate 1/*ra* + 1/*rb*. Look good? (NO), but it IS right.



Use [F2]: **expand(** and [F2]:**comDemon(** to simplify the expression. First, put the command down, then go up (▲)and retrieve ([ENTER]) the last result, then press [ENTER] to evaluate.

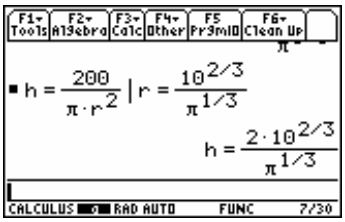
Capabilities of the TI-89

1



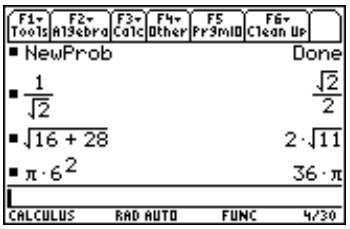
Numerical

2



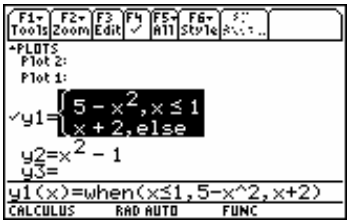
Volume of a can

3



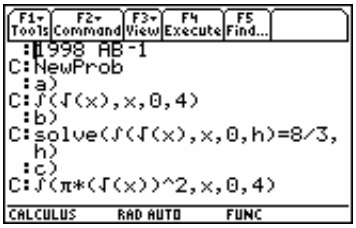
Symbolic answers

4



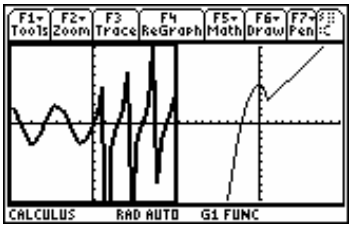
Piecewise graphing

5



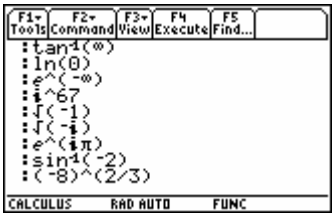
Scripting

6



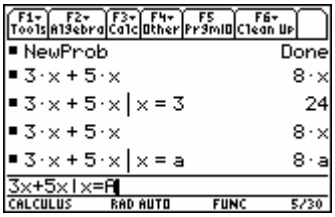
Two graphs

7



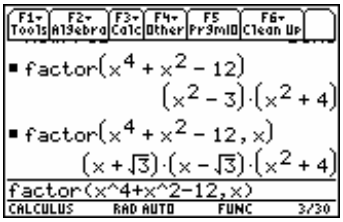
Special numbers

8



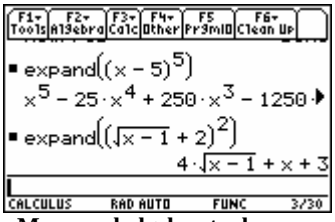
"simple" algebra

9



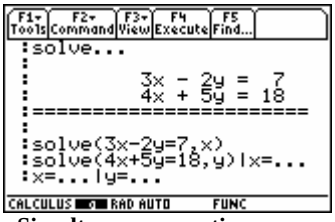
"better" algebra

10



More cool algebra tools

11



Simultaneous equations

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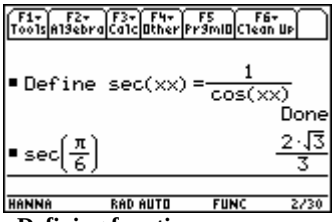

Defining functions