

Slope Fields Laboratory

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This document allows you to experiment with slope fields and first-order differential equations (DEs). A *slope field* is a graphical representation of the path of a function through the plane based on its DE. A *particular solution* is subject to an *initial condition* (IC) which determines a possible graph of the solution.

The DE can be subject to both x and y , and may be written in the form $y' = f(x, y)$. The path through the plane is not necessarily always parallel as in the 'differs by a constant' principle in the case of $y' = f(x)$ only DEs. See the sample, $dy/dx = x + y$.

Note that this document does not attempt to teach the techniques of solving the DE and relies on the `deSolve()` function to produce a graph of a particular solution $y(0) = c$.

Page 2 consists of a series of math boxes that

- * set up the domain and interval of the grid points of the slope field,
- * the differential equation,
- * the equation of the line segments that make up the slope field,
- * and the solution and initial condition of the DE.

Page 3 consists of two sliders and the graph box.

- * one slider (variable " k ") adjusts the length of the slope field segments
- * the other slider (variable " c ") adjusts the IC. The blue function graph changes accordingly

The **large print** math boxes ("Window" and "define DE") must be edited for your problem. If you enter invalid information simply go back and correct the mistake.

If the DE cannot be 'solved for y ', then the graph of the IC function will not appear.

Edit the window settings of the graph box on page 3 to fit the grid.

Enjoy,

John Hanna

Slope Field Investigation

This math box *defines* "Window()" to set up the size and spread of the grid: "Done"

Double-click the following call of the Window() function and enter xMin, xMax, xScl, yMin, yMax, yScl for the grid points of the slope field. Note that it does not adjust the 'viewing window' of the graph box on page 3.

Window(-5, 5, 2, -6, 6, 2)

This math box *defines* "Grid()" to set up the grid points: "Done"

This math box *calls* Grid() and sets up the grid points dataset for a stat plot. The output shows the number of grid points in the {horizontal, vertical} directions: {6, 7}

Here's the differential equation... double-click to edit:

define $d(x, y) = x + y$

This math box defines the slopes at each of the grid points...

listM := d(listX, listY)

This is the equation of the line through (a, b) with slope m...

define Line(a, b, m, x) = m.(x - a) + b

This math box determines the solution to the DE and displays the large red equation below. *f* is used by the graph box on page 3. If the IC $y(0) = c$ is not available (solution is *false*), then change this input to some other IC.

solve(deSolve(y' = d(x, y) and y(0) = c, x, y), y) → f

$$y = (c + 1) \cdot e^x - x - 1$$

NOTE: If you do not see $y = \langle \text{some function of } x \rangle$ above, then the **blue** graph of the solution will not appear on the graph on the next page.

This page lets you interact with the graph of the slope field and the solution of the DE.



Length of segments



Initial condition

Double-click the graph below and use the Zoom tools to view the grid...

