CYCLING ALONG THE YARRA

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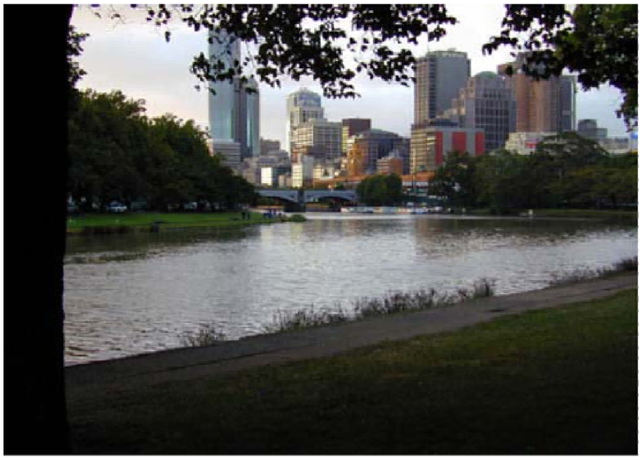

During recent months, bicycle tracks along the Yarra River have either been constructed or have been renewed. The picture shown on the right shows a track that follows the curve in the river. This is typical of many paths that follow the edge of the river.

In this investigation, you will use a variety of functions to model sections of a bicycle track. Surveyors mark out points the track must pass through. The curves can then be determined and mapped accordingly. As the paths are constructed in segments, so too the functions modelling the paths can be limited to model the region specified.

The figures provided in this investigation have been simplified in order reduce the complexity of the calculations.

a) Initially, consider the problem of determining a quadratic function whose curve passes through three points. Two of these points, *A* and *B*, have coordinates (1, 4) and (2, 2) respectively. The third point, *C*, has an *x*-value of 4 and is given by (4, *k*) where *k* is an arbitrary real constant.

(Write the quadratic equation in of the form: $f(x) = ax^2 + bx + c$)

i) With $k = 0$, use simultaneous equations to determine the function and sketch it on a suitable domain.

ii) With $k = 1$, use simultaneous equations to determine the function and sketch it on a separate set of axis over a suitable domain.

iii) Use simultaneous equations to determine *a, b* and *c* in terms of the parameter *k*.

iv) Systematically vary k and explore the effect of k on the behaviour of the quadratic functions produced. A number of graphs should be drawn on the same set of axes so that comparisons can be made. State clearly the effect that k has on the curves produced.

v) Experimentally determine the value of *k* that generates a linear graph. Use your answer from part a (iii) to explain why the graph is linear.

vi) Determine the range of values for *k* that give a positive x^2 coefficient.

vii) Determine the range of values for *k* that give a negative x^2 coefficient.

b) The next section of the bicycle track is to be modelled by a second quadratic equation. The next section of the track passes through the points D (8,0), E (12,-7) and joins the first section at C (4,k). (Write the quadratic equation in of the form: $g(x) = ax^2 + bx + c$)

i) With k = 0 determine the equation to the new section of the path using simultaneous equations.

ii) With k = 0, sketch the first section of the path, passing through points A, B and C on the same set of axis as your new graph passing through C, D and E. Comment on the way the paths join at point C.

iii) With k = 1 determine the equation to the new section of the path using simultaneous equations.

iv) With k = 1, sketch the first section of the path, passing through points A, B and C on the same set of axis as your graph passing through C, D and E. Comment on the way the paths join at point C.

v) Use simultaneous equations to determine a, b and c in terms of the parameter k.

vi) Systematically vary k and explore the effect of k on the behaviour of both quadratic functions produced. A number of graphs should be drawn on the same set of axes so that comparisons can be made. What do you notice about the way the graphs join at point C.

vii) Experimentally determine a value of k so that the graphs join smoothly. Comment on how you determined this value of k.

viii) Show that for your value of k the graphs join smoothly. (Differential calculus is not required for this answer.)

Cycling along the Yarra

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Part a)

define $f(x) = a \cdot x^2 + b \cdot x + c$

L1	L2
1	4
2	2
12	-7
8	0

$f(1) = 4 \quad \Rightarrow a + b + c = 4$

$f(2) = 2 \quad \Rightarrow 4 \cdot a + 2 \cdot b + c = 2$

$f(4) = k \quad \Rightarrow 16 \cdot a + 4 \cdot b + c = k$

a) iii)

solve$(f(1) = 4$ and $f(2) = 2$ and $f(4) = k, \{a, b, c\})$

$$a = \frac{k+2}{6} \text{ and } b = \frac{-(k+6)}{2} \text{ and } c = \frac{k+20}{3}$$

$\frac{k+2}{6} \rightarrow a \quad = \frac{k+2}{6}$ $\quad \frac{-(k+6)}{2} \rightarrow b \quad = \frac{-(k+6)}{2}$ $\quad \frac{k+20}{3} \rightarrow c \quad = \frac{k+20}{3}$

Part b)

define $h(x) = d \cdot x^2 + e \cdot x + g$

$h(12) = -7 \quad \Rightarrow 144 \cdot d + 12 \cdot e + g = -7$ $\qquad h(8) = 0 \quad \Rightarrow 64 \cdot d + 8 \cdot e + g = 0$

$h(4) = k \quad \Rightarrow 16 \cdot d + 4 \cdot e + g = k$

solve$(h(12) = -7$ and $h(8) = 0$ and $h(4) = k, \{d, e, g\})$

$$d = \frac{k-7}{32} \text{ and } e = \frac{-(5 \cdot k - 21)}{8} \text{ and } g = 3 \cdot k - 7$$

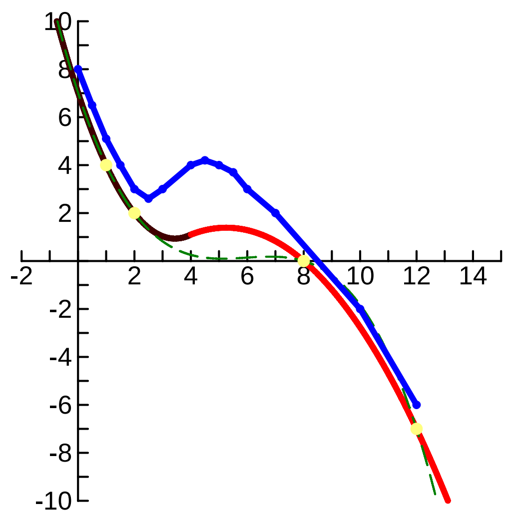
$\frac{k-7}{32} \rightarrow d$ $\qquad \frac{21 - 5 \cdot k}{8} \rightarrow e$ $\qquad 3 \cdot k - 7 \rightarrow g$

b) v)

This is an interesting point to solve for k. When the two graphs join smoothly there will be only one point of intersection. Therefore the second point of intersection must also give x = 4.

solve$(f(x) = h(x), x) \quad \Rightarrow x = \frac{-8 \cdot (8 \cdot k - 41)}{13 \cdot k + 53}$ or $x = 4$

solve$\left(\frac{-8 \cdot (8 \cdot k - 41)}{13 \cdot k + 53} = 4, k\right) \quad \Rightarrow k = 1$



Cubic Regression

regEQ(x) = -.034524x^3 + .617857x^2 + -3.6119x + 7.02857

value of k 1.1

$\text{solve}(f(x) = h(x), x)$

$x = 4.$ or $x = 3.82764$