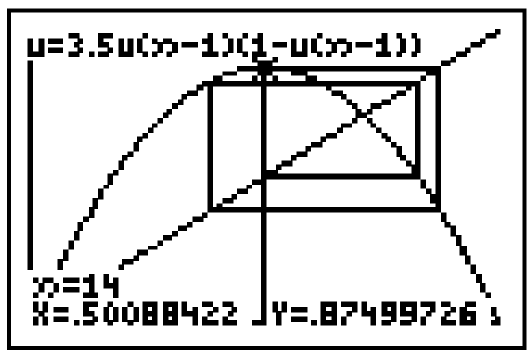
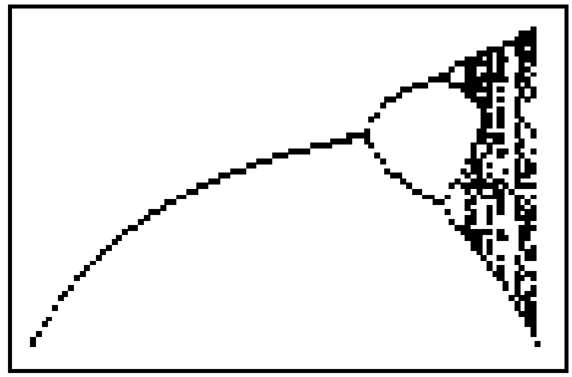
Robert Iovinelli's article on chaotic behavior in the Feb. 2000 *Mathematics Teacher* was refreshing. Here's the rest of the story…

1. In the TI-82/3/3+, there's a **Web Plot** setting that really illustrates the behavior of this iteration and other recursive patterns. When in **Sequence** mode, on your **Format** (press [2nd][ZOOM]) menu, there's a setting for **Web** or **Time** plot. Switch to **Web** and set the viewing window to [0,1]x[0,1]. Press [GRAPH]. This produces the parabola $y = ax(1-x)$ with the line $y = x$ through it. Press [TRACE]. Press [▶]. Every *two* presses of [▶] is one more step in the iteration. The second press jumps to the line $y = x$ because, in order to calculate the next term in the sequence, the calculator must first make the current value of $y$ the new value of $x$. Using different values for $a$ and tracing graphically illustrates the way that different numbers of attractors can occur. The figure at the right illustrates the four attractors that occur when $a = 3.5$.




2. To finish the story, it is necessary to examine the possible values of $a$ and the impact that they have on the number of attractors. A program is necessary here. The following calculator program produces the **Feigenbaum diagram** illustrating the transition from order to chaos and the intermittent bursts of order on the way! It will work on the TI-73, 82, 83, and 83+ and the same structure will work on all the other calculators. First, set your viewing window to [1,4]x[0,1].

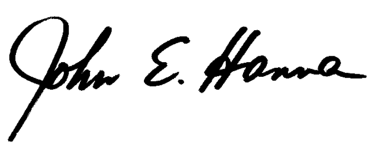
```
PROGRAM:FEIGEN
FnOff
PlotsOff
ClrDraw
For(a,xmin,xmax,Δx)
   Ø.5→x
   For(i,1,4Ø,1)
      a*x*(1-x)→x
   EndFor
   For(i,1,8Ø,1)
      a*x*(1-x)→x
      PtOn(a,x)
   EndFor
EndFor
```



Notes on the program:

- The **For(a** loop goes across the screen and uses these as values for *a* in the expression *ax(1-x)*. Values outside [1,4] cause errors. Why?
- 0.5 is the initial value of the sequence. Does it matter?
- The first **For(i** loop computes the first 40 terms of the sequence.
- The second **For(i** loop computes the next 80 terms and plots their values on the screen.

For more information on Period-Doubling and Chaos, see Peitgen, Jurgens, Saupe, **Fractals for the Classroom**, vol. 2, ch. 11, Springer-Verlag, 1992, available from NCTM.



*John Hanna*
*$T^3$ – Teachers Teaching with Technology*
*Teaneck High School*
*100 Elizabeth Ave.*
*Teaneck, NJ 07666*
*tejohhan@bergen.org*
*www.bergen.org/~tejohhan*